Ant Colony Optimized Importance Sampling: Principles, Applications and Challenges

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Extended abstract

Importance Sampling (IS) is an efficient rare event simulation technique provided that an appropriate *change of measure* can be obtained. It is particularly useful in practice if a good, adaptive change of measure can be determined "automatically" for a broad class of models without requiring excessive mathematical pre-analysis of the specific model under consideration. Many of the adaptive approaches reported in the literature aim at directly minimizing the (estimated) variance of the IS estimator. An alternative approach is the cross entropy method which aims at minimizing the cross entropy between the currently used measure and the (unavailable) optimal measure. Unfortunately, huge storage requirements often limit the method to quite small models, for example when considering general state dependent change of measure strategies for Markovian models. Larger models can be handled by restriction to a state independent change of measure but this works well for some systems only.

Ant Colony Optimized Importance Sampling (ACO-IS), first presented in [4], is an adaptive approach to determine a state dependent change of measure via Ant Colony Optimization (ACO) [1,2], a metaheuristic for solving combinatorial optimization problems that is inspired by the foraging behavior of real ants. The artificial ant system is a multi-agent system where the ants search iteratively for the optimal solution. Applied in communication network routing the ants search for the best (optimal) path in a connected graph (network) between the source and destination nodes. Ants are sent from the source node ("the ant nest") and search randomly to find the destination ("the food"). The path quality is evaluated on arrival to the destination node and then the ants return to the source along the reversed path leaving markings ("pheromones") along the reverse path to guide future ants in their search for the same destination. The better the path, the stronger the pheromone updates. ACO-IS for Markovian models applies a similar approach to gradually let the IS change of measure adapt to the current model. The nodes are states in the Markov model, and the links correspond to state transitions. The source nodes are the origin or regenerative states, and the destination nodes are given as the set of rare events of interest. The path quality in ACO-IS that determines the markings of a specific transition is the accumulated path probability (potentially multiplied by the importance of the destination node) given that this specific transition was the first step. This is repeated for all transitions along the recorded path. The overhead of ACO-IS in terms of memory is storage of markings at each transition per *visited* state. Initially there are no markings associated with the transitions, and then the ants may either perform a (guided) random walk, or exploit knowledge for similar models, if available.

ACO-IS has been already successfully applied to rare events with regard to performance, reliability and quality of service (QoS) properties in different optical network architectures [4,5,6]. The simulation results from all these examples have demonstrated the accuracy of the obtained estimates. Details of the adapted change of measure have been investigated in [3] and provided first insights into the inner workings of the method. In these early applications, exponentially distributed times and models with lattice-structured transition diagrams of the underlying multi-dimensional Markov chains are considered where each transition only changes one component of the state. But ACO-IS is not restricted to this model class. With the same underlying rationale it is extended to more general structures, and phase-type distributed times are incorporated.

ACO-IS is young, research on it is still at an early stage and a couple of challenging issues require further investigation. Analyzing the efficiency and establishing convergence properties should be the ultimate goal for any adaptive algorithm. Comparative studies with other adaptive approaches are the first steps towards figuring out strengths and weaknesses, pros and cons, and the method's competitiveness. Further research also includes systematic studies of the asymptotic robustness of ACO-IS and the importance of the initial change of measure.

References

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